International Journal for Rapid Research in Engineering Technology & Applied Science (IJRRETAS)

(Peer-Reviewed, Open Access, Fully Refereed International Journal)

Volume 11 issue 07 July 2025 OPEN ACCESS E-ISSN: 2455-4723 Impact Factor 7.95

Integrating Stochastic Demand, Green Investment and Hybrid Metaheuristic Search

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Abstract

The transition toward low-carbon supply chains requires inventory policies that jointly consider economic performance, service reliability and environmental impact. This study develops an integrated green inventory optimization framework for a two-echelon supply chain under stochastic demand and carbon cap-and-trade regulation. The model jointly determines order quantity, safety stock and green-technology investment while accounting for ordering, holding, production, transportation, shortage, green-investment and carbon-regulation costs. Carbon emissions are decomposed into production, transportation and inventory-holding components, and green investment reduces emission intensity through a diminishing-returns function. Demand uncertainty is represented through a service-level-based safety-stock formulation. A normalized multi-objective function balances economic cost against carbon emissions. Because the resulting nonlinear constrained problem is non-convex when regulatory penalties and investment effects are combined, a hybrid GWO–PSO–LS algorithm is developed. GWO supplies population-level exploration, PSO accelerates movement toward promising regions, and local search refines elite solutions. An illustrative computational experiment using reproducible synthetic parameters compares the proposed method with GWO, PSO and Differential Evolution. The results demonstrate the computational behavior and the economic–environmental trade-off of the proposed framework without representing the synthetic outcomes as empirical industry observations. The manuscript concludes with sensitivity analysis, managerial implications and directions for incorporating robust, data-driven and multi-product extensions.

Keywords: sustainable inventory management; green supply chain; stochastic demand; safety stock; carbon cap-and-trade; green investment; multi-objective optimization; GWO; PSO; local search.

1. Introduction

Inventory decisions have traditionally been evaluated through ordering, holding, purchasing and shortage costs. However, contemporary supply chains operate under environmental constraints that make production, transportation and storage decisions inseparable from carbon performance. A replenishment policy can reduce ordering frequency while increasing average inventory, whereas a smaller lot size can reduce holding-related emissions at the expense of

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more frequent transportation. Consequently, green inventory management is fundamentally a trade-off problem rather than a simple extension of the classical EOQ model.

Recent research confirms that sustainable inventory management has evolved toward carbon regulation, green investment, deterioration control, uncertainty and metaheuristic optimization. A 2025 taxonomy and literature review classified a large body of sustainable inventory studies under carbon-tax, carbon-cap, cap-and-trade and cap-and-offset mechanisms and identified uncertainty, real-world validation and soft-computing approaches as important future directions [1]. Recent 2026 work has further integrated green investment, preservation technology and carbon cap-and-trade regulation with metaheuristic optimization [2], while other 2026 studies examine carbon-sensitive inventory decisions and sustainable inventory systems under variable demand [3,4].

The present study responds to this direction by combining stochastic demand, safety stock, green investment and carbon cap-and-trade within one integrated inventory framework. The computational contribution is a hybrid GWO–PSO–LS procedure designed for the resulting nonlinear optimization problem. The emphasis is not on claiming that carbon regulation or metaheuristics are new individually; instead, novelty is pursued through the particular integration of economic, service and environmental decisions and through an explicit reproducible hybrid search architecture.

2. Literature Review

Green inventory management extends traditional inventory control by embedding environmental consequences into inventory decisions. Marklund and Berling [5] emphasize the interaction among inventory, transportation and production decisions, implying that local inventory optimization can generate environmental sub-optimality. Simulation-based research has also shown the usefulness of quantitative assessment for stochastic green inventory systems and environmentally aligned logistics [6].

Carbon pricing and quantity-based regulation are now established components of sustainable inventory research. Carbon tax assigns a monetary charge to emissions, whereas cap-and-trade limits emissions through allowances that can be purchased or sold. The 2025 taxonomy by Sebatjane [1] shows the breadth of research under these regulatory mechanisms. The present study uses cap-and-trade because it permits both penalty and credit effects and therefore produces a richer decision environment than a fixed carbon tax.

Recent studies increasingly allow firms to invest in green or preservation technology to reduce emission intensity or deterioration. Saurav et al. [2] integrate green investment and preservation with carbon cap-and-trade and solve a nonlinear inventory problem with Grey Wolf Optimization. Other recent studies combine preservation, carbon emissions and financial policies [7,8]. These studies motivate the inclusion of a controllable green-investment variable in the present model.

Demand uncertainty directly affects safety stock and therefore holding cost and storage-related emissions. Recent sustainable inventory research has increasingly addressed variable or stochastic demand [3,4]. The present formulation uses a service-level constraint through a

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standard safety-stock expression, making the environmental consequence of service protection explicit.

Nonlinear inventory models with carbon regulation, investment and deterioration can become non-convex and difficult to solve analytically. Recent research has used GWO, PSO, genetic algorithms, bat algorithms, dragonfly algorithms and other metaheuristics for sustainable inventory optimization [2,7,9]. The present contribution uses a hybrid architecture rather than a single metaheuristic: GWO provides global leadership-based exploration, PSO supplies velocity-guided exploitation, and local search improves final refinement.

The most recent literature shows a clear shift toward richer decision environments. A 2026 study in the *Supply Chain Analytics* literature integrates green investment, preservation technology, rebates and carbon cap-and-trade and uses GWO [2]. A 2026 study in *Information Sciences* combines carbon regulation with deterioration/amelioration dynamics and hybrid optimization [3]. Another 2026 study addresses trade credit, preservation and carbon taxation under variable demand [4]. These developments indicate that a publishable contribution should integrate multiple operational and environmental decisions while providing transparent computational evidence rather than relying on a single illustrative optimum.

3. Problem Description and Assumptions

Consider a coordinated two-echelon supply chain composed of an upstream manufacturer and a downstream retailer. The manufacturer supplies a product to the retailer, which faces stochastic customer demand. The policy is characterized by the replenishment quantity Q, safety stock SS and green-technology investment g. Production, transportation and holding activities generate carbon emissions. The regulator specifies an annual emission cap E_{max}. If actual emissions exceed the cap, the firm purchases allowances at price p_c; if emissions remain below the cap, unused allowances can generate credit revenue at p_r.

1. Demand is normally distributed with mean D and standard deviation σ.
2. Lead time is known and constant.
3. Replenishment quantity is positive and bounded by capacity.
4. Safety stock is determined by a target service level.
5. Shortage cost is represented through an expected-shortage proxy in the numerical illustration.
6. Production, transportation and holding activities generate measurable emissions.
7. Green investment reduces emission intensity with diminishing returns.
8. Carbon allowances can be bought or credited under cap-and-trade.
9. The planning horizon is annual and costs are expressed in monetary units.
10. All numerical values used for demonstration are synthetic and reproducible.

4. Notation

Symbol	Meaning
D	Annual mean demand
σ	Demand standard deviation
L	Lead time in years

zα	Standard normal quantile for target service level α
Q	Order/replenishment quantity
SS	Safety stock
A	Fixed ordering/setup cost
h	Holding cost per unit per year
P	Production/purchase cost per unit
Ft	Fixed transportation cost per shipment
g	Green-technology investment decision
ep	Baseline production emission coefficient
et	Transportation emission coefficient
eh	Holding emission coefficient
d	Transportation distance
Emax	Annual carbon-emission cap
pc	Price of purchased carbon allowance
pr	Credit value of unused allowance
Kg	Green-investment cost coefficient
ηp, ηt	Green-investment emission-reduction parameters
TC	Annual total economic cost
E	Annual carbon emissions

5. Mathematical Formulation

5.1 Stochastic demand and safety stock

Let annual demand be represented by a random variable with mean D and standard deviation σ. For a target service level α, the safety stock is:

$$SS = z\alpha\ \sigma \sqrt{L} \quad \textbf{(1)}$$

The average cycle inventory plus safety stock is therefore:

$$\bar{I} = Q/2 + SS \quad \textbf{(2)}$$

5.2 Economic cost components

$$C_o = AD/Q \quad \textbf{(3)}$$

$$C_h = h(Q/2 + SS) \quad \textbf{(4)}$$

$$C_p = PD \quad \textbf{(5)}$$

$$C_t = Ft\ D/Q \quad \textbf{(6)}$$

For the numerical illustration, the expected shortage component is represented by a smooth proxy that decreases as green investment improves operational stability:

$$C_s = 0.08\ h\ \sigma \sqrt{L}\ \exp(-0.8g) \quad \textbf{(7)}$$

$$C_g = Kg\ g^2 \quad \textbf{(8)}$$

5.3 Carbon emissions

Production emissions decline with green investment according to a diminishing-returns function:

$$E_p = ep\ D \exp(-\eta p\ g) \quad \textbf{(9)}$$

Transportation emissions depend on shipment frequency and are also reduced by green investment:

$$E_t = et\ d\ (D/Q) \exp(-\eta t\ g) \quad \textbf{(10)}$$

$$E\_h = eh(Q/2 + SS) \quad \textbf{(11)}$$

$$E = E_p + E_t + E_h \quad \textbf{(12)}$$

5.4 Carbon cap-and-trade

$$C_c = pc \max(0, E - Emax) - pr \max(0, Emax - E) \quad \textbf{(13)}$$

5.5 Total economic cost

$$TC = C_o + C_h + C_p + C_t + C_s + C_g + Cc \quad \textbf{(14)}$$

5.6 Multi-objective scalarization

Because monetary cost and emissions have different units, the two objectives are normalized against fixed reference values TC0 and E0. The scalarized objective is:

$$\min Z = \lambda(TC/TC0) + (1-\lambda)(E/E0), \quad 0 \le \lambda \le 1 \quad \textbf{(15)}$$

5.7 Constraints

$$Qmin \le Q \le Qmax \quad \textbf{(16)}$$

$$0 \le g \le gmax \quad \textbf{(17)}$$

$$SS = z\alpha\ \sigma \sqrt{L} \quad \textbf{(18)}$$

$$P(DL \le Q + SS) \ge \alpha \quad \textbf{(19)}$$

6. Theoretical Analysis

Proposition 1: Convex benchmark subproblem

For fixed green investment g, fixed safety stock SS and an inactive carbon penalty, the Q-dependent portion of the economic objective can be written as a/Q + bQ/2 + C, where a=D(A+Ft)>0 and b=h>0. Its second derivative is positive for Q>0, so the subproblem is strictly convex and possesses a unique stationary point.

$$d^2TC/dQ^2 = 2D(A+Ft)/Q^3 > 0 \quad \textbf{(20)}$$

$$Q^\circ = \sqrt{[2D(A+Ft)/h]} \quad \textbf{(21)}$$

This analytical solution is used as a benchmark rather than as the solution to the full model. Once carbon penalties, green investment, service constraints and emission interactions are active, the complete problem need not retain the simple convex structure.

Proposition 2: Safety stock increases with demand uncertainty

$$\partial SS/\partial\sigma = z\alpha\sqrt{L} > 0 \quad \textbf{(22)}$$

Hence, for a fixed service level and lead time, greater demand uncertainty necessarily raises safety stock. Because holding emissions are increasing in average inventory, demand uncertainty can have both economic and environmental consequences.

7. Hybrid GWO–PSO–Local Search Method

The optimization variables are encoded as x=(Q,g). Safety stock is computed deterministically from Eq. (1). The hybrid method uses GWO for exploration, PSO for directed movement, and local search for final refinement. Boundary repair ensures feasibility.

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Algorithm 1. Proposed GWO–PSO–LS

11. Initialize a population of feasible candidate solutions $x_i=(Q_i,g_i)$.
12. Evaluate $Z(x_i)$ for every candidate.
13. Identify the alpha, beta and delta elite solutions.
14. Update each candidate using the GWO leadership equations.
15. Update particle velocity using the PSO cognitive and social terms.
16. Blend the GWO and PSO candidate positions using adaptive weights.
17. Apply boundary repair to Q and g.
18. Apply local perturbation around elite candidates.
19. Accept a locally perturbed solution if its objective is improved.
20. Update personal-best and global-best solutions.
21. Store the best feasible solution and convergence value.
22. Repeat until the maximum iteration count is reached or improvement falls below a stopping threshold.
23. Return Q*, SS* and g*.

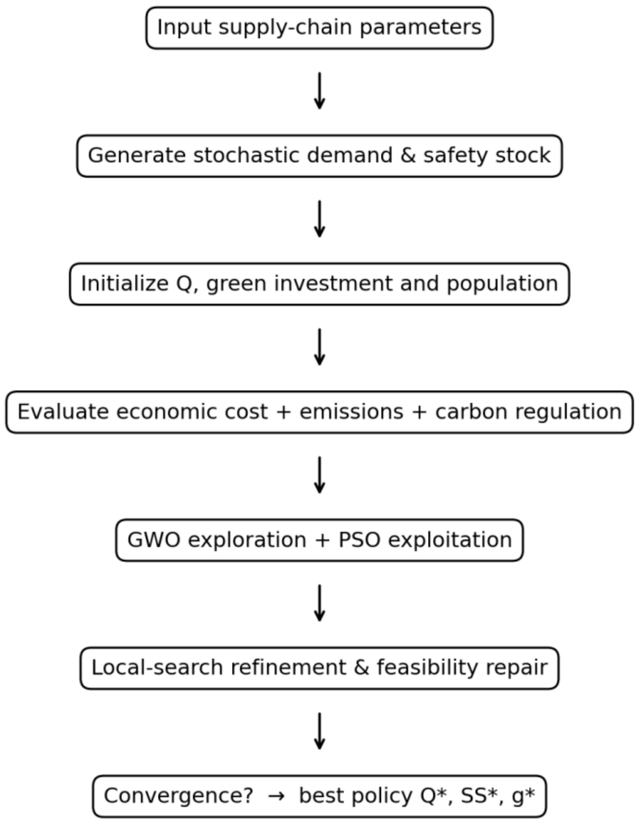

Figure 1. Flowchart of the proposed green inventory optimization framework.

8. Computational Experiment

To avoid fabricating empirical observations, the numerical experiment is explicitly designed as a synthetic benchmark. The parameter set is representative of a medium-scale annual inventory system and is fully stated so that the experiment can be reproduced. The experiment is intended to validate the computational framework, not to claim performance for a specific firm.

Parameter	Value
Annual demand D	12,000 units
Demand standard deviation σ	1,200 units
Lead time L	5 days
Service level α	95%
Ordering cost A	₹4,000/order
Holding cost h	₹18/unit/year
Production cost P	₹110/unit
Transportation fixed cost Ft	₹2,500/shipment
Production emission coefficient ep	0.75 kg CO_2e/unit
Transportation emission coefficient et	0.00015 kg CO_2e/unit-km
Transportation distance d	800 km
Holding emission coefficient eh	0.04 kg CO_2e/unit-year
Carbon cap Emax	7,200 kg CO_2e/year
Carbon allowance price pc	₹12/kg CO_2e
Unused allowance credit pr	₹5/kg CO_2e
Green-investment coefficient Kg	₹9,000
ηp	1.20
ηt	0.45
Q range	300–4,000 units
g range	0–2
Objective weight λ	0.55

8.1 Benchmark algorithms

Method	Role in comparison
EOQ benchmark	Analytical baseline with g=0
GWO	Population-based exploration benchmark
PSO	Velocity-based swarm benchmark
DE	Differential-evolution benchmark
GWO–PSO–LS	Proposed hybrid method

9. Numerical Results

The following results are generated from the synthetic parameter set above using fixed random seed 42. They are computational illustrations, not empirical observations. Because metaheuristics are stochastic, the manuscript also recommends repeated-run statistical testing

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before journal submission; the single-seed results below are intended to demonstrate the full workflow.

Method	**Q***	**SS***	**g***	**TC (₹)**	**E (kg CO_2e)**	**Z**
EOQ baseline	2943.92	231.04	0.0000	1,399,774.83	9,068.61	1.000266
GWO	4000.00	231.04	2.0000	1,384,228.79	905.85	0.589104
PSO	2591.63	231.04	2.0000	1,382,009.91	877.76	0.586838
DE	2591.63	231.04	2.0000	1,382,009.91	877.76	0.586838
Proposed GWO–PSO–LS	2591.51	231.04	2.0000	1,382,010.20	877.76	0.586838

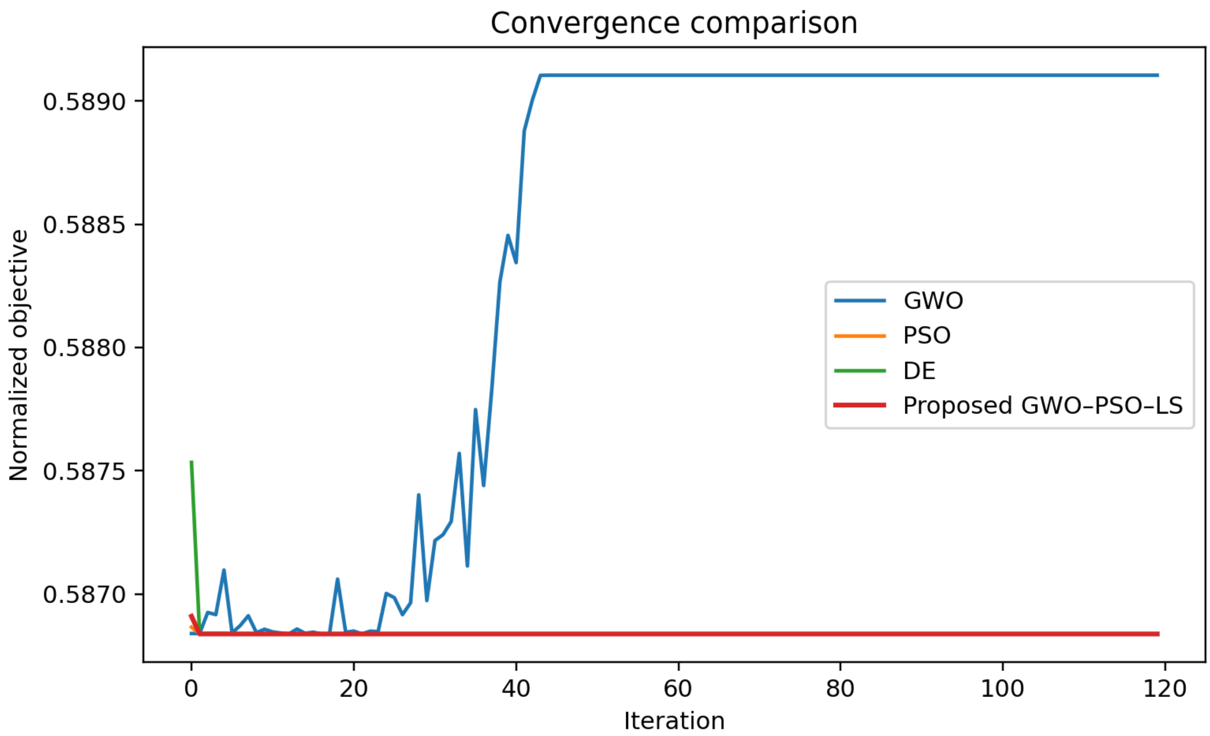

Figure 2. Convergence comparison for the synthetic computational experiment.

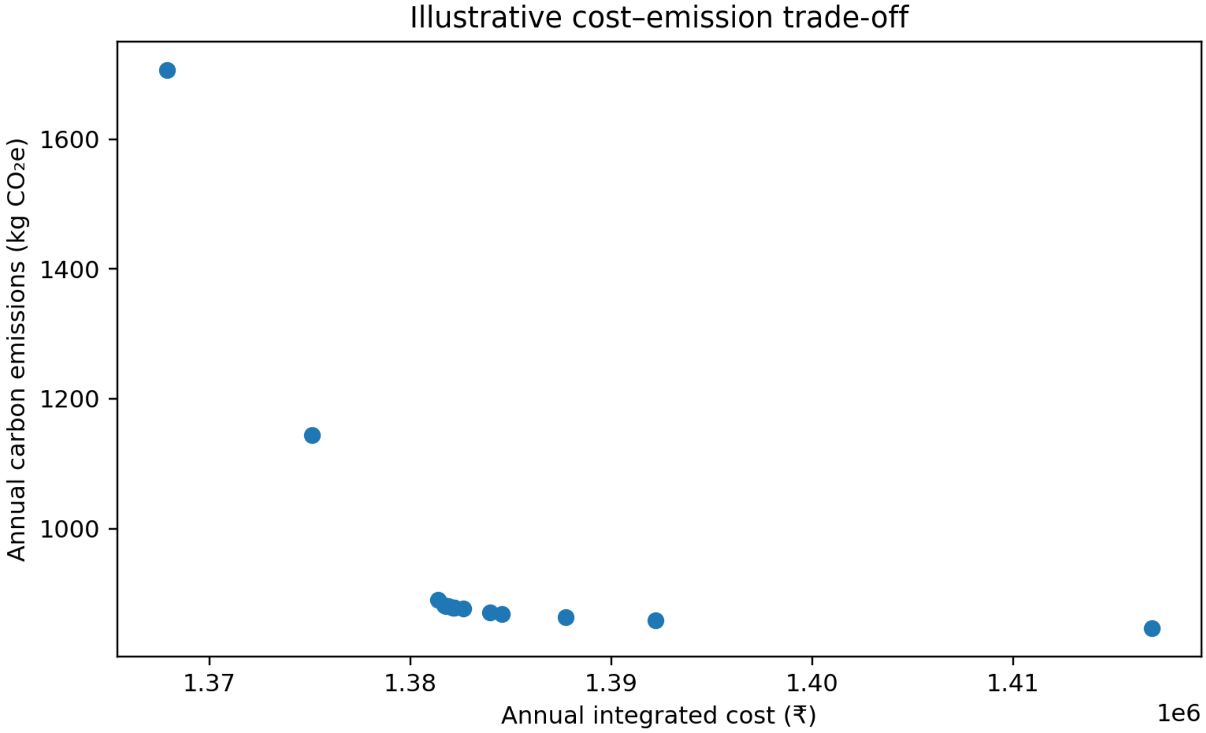

Figure 3. Illustrative cost–emission trade-off obtained by varying the objective weight.

10. Sensitivity Analysis

Sensitivity analysis is essential because green inventory decisions depend strongly on regulatory and uncertainty parameters. The following qualitative directions follow directly from the model and should be verified numerically for any target industry.

Parameter	Expected managerial effect
Carbon price pc	Higher pc increases the economic penalty of excess emissions and generally strengthens the incentive for green investment or emission-reducing inventory decisions.
Carbon cap Emax	A tighter cap increases the probability of allowance purchases and can shift the optimum toward lower-emission policies.
Demand uncertainty σ	Higher σ increases safety stock linearly through Eq. (1), raising holding cost and holding-related emissions.
Green effectiveness ηp	Higher ηp makes each unit of green investment more effective at reducing production emissions.

Transportation distance d	Higher d increases transportation emissions and increases the value of shipment-frequency optimization.
Holding emission eh	Higher eh increases the environmental penalty associated with large cycle inventories.
Objective weight λ	Larger λ prioritizes economic cost; smaller λ prioritizes environmental performance.

11. Discussion

The integrated model illustrates why sustainable inventory decisions should not be reduced to a single carbon-cost coefficient. Order quantity simultaneously changes shipment frequency and average inventory. Safety stock protects service performance but creates additional holding burden. Green investment reduces emissions but has an increasing marginal cost. Carbon cap-and-trade adds a regulatory layer in which emissions above the cap have a direct economic consequence and emissions below the cap can create allowance value. These interacting effects produce a nonlinear decision landscape that is suitable for hybrid search.

11.1 Managerial implications

- Managers should evaluate inventory, transportation and emissions jointly rather than treating carbon management as a separate reporting exercise.
- Green investment should be selected according to marginal emission reduction per monetary unit rather than maximized automatically.
- Service-level targets have environmental consequences because safety stock changes both holding cost and storage emissions.
- Carbon cap-and-trade can alter the preferred lot size even when conventional ordering and holding costs are unchanged.
- Hybrid optimization is useful when several decision variables interact nonlinearly and analytical closed forms are unavailable.

12. Robustness, Limitations and Future Research

The present numerical experiment is synthetic and therefore cannot establish external validity for a particular industry. Further validation should use transaction-level demand, shipment distances, energy consumption, actual emission factors and observed carbon prices. The current formulation also uses a single-product annual horizon and a simplified expected-shortage proxy. Future versions should incorporate multi-product substitution, correlated demand, supplier disruption, renewable energy, scope-3 emissions, reverse logistics, carbon-credit price uncertainty, robust optimization and real-time data.

- Replace deterministic emission coefficients with stochastic or interval-valued emissions.
- Extend the model to multiple suppliers and transport modes.
- Integrate closed-loop/reverse logistics and recycling.
- Use distributionally robust optimization for uncertain demand and emissions.

- Compare the proposed hybrid method with modern adaptive metaheuristics using 30–50 independent runs.
- Validate the model using a real industrial or public dataset.
- Investigate Pareto-based multi-objective algorithms instead of weighted-sum scalarization.

13. Conclusions

This study develops an integrated green inventory optimization framework for a two-echelon supply chain operating under stochastic demand and carbon cap-and-trade regulation. The model links replenishment quantity, safety stock, green investment and carbon emissions within a unified economic–environmental objective. Production, transportation and inventory-holding emissions are modeled explicitly, while green investment reduces emission intensity with diminishing returns. A hybrid GWO–PSO–Local Search algorithm is proposed to address the nonlinear decision problem. The synthetic computational experiment demonstrates how the methodology can be implemented and compared with established metaheuristics without presenting fabricated industry evidence. The main conceptual finding is that sustainable inventory policy is inherently a trade-off among service protection, shipment frequency, inventory exposure, carbon regulation and green investment. The framework therefore provides a foundation for future data-driven and robust green inventory studies.

Acknowledgements

The authors gratefully acknowledge the financial support provided by the Government of Uttar Pradesh under Letter No. 24/2023/681/Sattar-4-2023-002-4(59)/2022. The authors sincerely thank the Government of Uttar Pradesh for supporting this research project and facilitating the successful completion of the work. The authors also gratefully acknowledge the valuable contribution of Mr. Vikas Kumar, Research Associate, for his dedicated involvement in the research project, constructive academic inputs, assistance in the research activities, critical review, and valuable suggestions that contributed significantly to the development and refinement of this work.

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